MEDICAL UNIVERSITY – PLEVEN
FACULTY OF MEDICINE

DIVISION OF PHYSICS AND BIOPHYSICS

LECTURE 10

ELASTICITY AND WAVE MOTION

Elasticity. Periodic motion and resonance. Traveling waves. Wave properties of sound and light. Energy in waves. Interference and standing waves. The Doppler effect. Ultrasonic sound

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When a spring or a rubber band is stretched, it tends to return to its original length when released, thereby demonstrating *elasticity*.

Def. Elasticity is the property of an object to restore its original dimensions after the distorting forces are removed.

Most solids regain their shape after small distortions. If the distorting forces exceed the elastic limit, then a permanent deformation will occur.

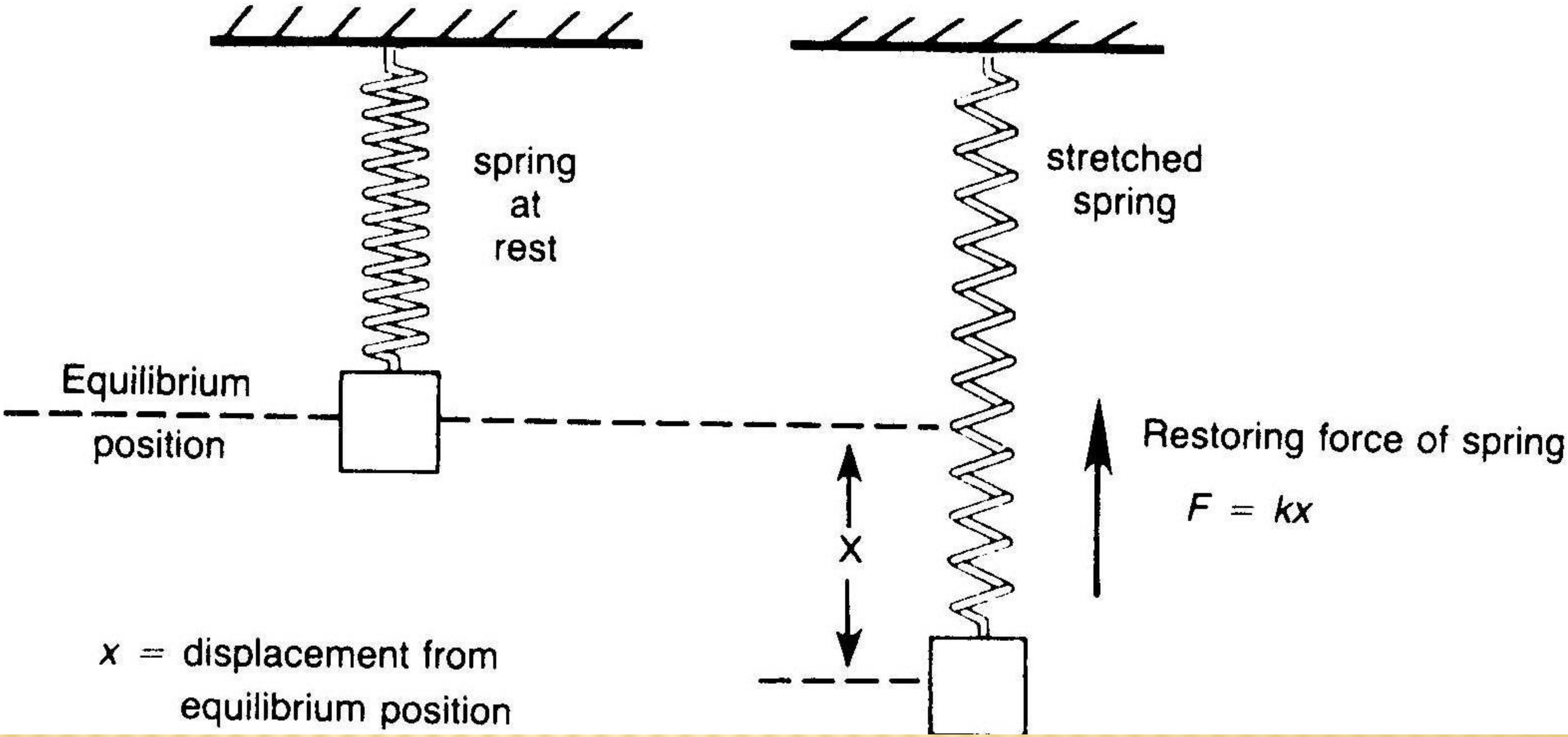
One substance is said to be more elastic than another if it returns to its original shape with greater precision.

Elasticity is not necessarily correlated with ease of deformation.

Within the elastic limit, the deformation of an elastic solid is proportional to the deforming force. This linear relationship is called **Hooke's law**.

For one-dimensional motion like the stretching of a spring, Hooke's law can be written $F = kx$, where F is the deforming force, x is the amount of stretch, and k is a numerical constant.

For a spring, k is called the stiffness constant and increases with stiffness.


spring
at
rest
stretched
spring
Equilibrium
position
Restoring force of spring
F = kx
X
x = displacement from
equilibrium position

PERIODIC MOTION AND RESONANCE

If a spring is hung vertically and a mass is suspended from the end of it, the spring will stretch until the elastic restoring force is great enough to support the weight of the object.

If the mass is pulled downward and released, it will oscillate up and down in “a simple harmonic motion".

It cannot settle immediately to rest at the equilibrium point because of E which has been given to it. It will move up and down periodically until all that E has been dissipated into internal energy (disordered molecular motion in the spring and surrounding air).

Periodic motion is a characteristic of elastic objects moving under the influence of Hooke's forces. Even on the molecular level, the atoms in solids vibrate in an approximation of simple harmonic motion.

Periodic motion is defined as motion which repeats itself at regular time intervals.

$$y(t) = A \sin(\omega t - \varphi)$$

T - the time required to complete one full cycle

f - is the number of cycles completed per second:

f=1/T [s^{-1}] or [Hz].

Def. The *amplitude* of the motion is the maximum distance moved from the equilibrium point.

Periodic motion will occur at one or more “natural frequencies” or “resonant frequencies” that are determined by the physical properties of the system such as mass, length, stiffness, ect.

A mass on a spring has only one natural frequency, determined by the mass and the stiffness of the spring according to the relationship.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

An important aspect of resonance is the fact that a system will tend to pick out its natural or resonant frequency.

If you give a child on a swing a simple push in one direction, the swing will then move at the natural frequency of the pendulum that forms the swing. This resonant frequency is so reproducible that it has been used in clocks for centuries.

Another feature of resonance is that it is easy to add energy to the system at its natural frequency, hard at other frequencies.

It is easy to get the child's swing moving through a large arc with periodic gentle pushes timed to coincide with the swing's natural frequency.

The mass on the spring, the pendulum, the guitar string, the air column of the trumpet – they all have their natural frequencies.

The electrical resonance of the receiving circuitry in our radio allows us to select the station we wish to listen to, while rejecting the dozens of other frequencies that strike our antenna at the same time.

Large scale resonances determine the length of our year, and small scale resonances determine the colors of light coming from a brightly hued object.

Resonances play a part in the structure of atoms and their chemical properties.

TRAVELING WAVES

If any part of an elastic object is disturbed, the disturbance will tend to propagate to all parts of the object.

E.g. if one end of a stretched wire is struck, the resulting vibration will travel to all parts of the wire.

Any collection of matter which has a definite equilibrium state will have a tendency toward periodic motion, and in an extended body that periodic motion will take the form of a traveling wave.

E.g. - the surface of a quiet pond when a pebble is dropped into it. Waves will move out from the disturbance in widening circles.

Transverse Waves - the particles of the medium are oscillating back and forth perpendicular to the direction of propagation of the wave.

Waves in a string or rope are examples of transverse waves.

Although light waves do not involve the movement of a material medium to propagate them, they as well as radio waves, X-rays, ect. are also transverse waves.

Longitudinal Waves - the periodic motion of the particles is parallel to the propagation direction.

The wave motion propagates as a series of contractions and expansions of the spring. The individual particles of the spring are undergoing simple harmonic motion about their equilibrium points, the points where they would be if the spring were at rest.

Sound waves in air are **longitudinal waves.**

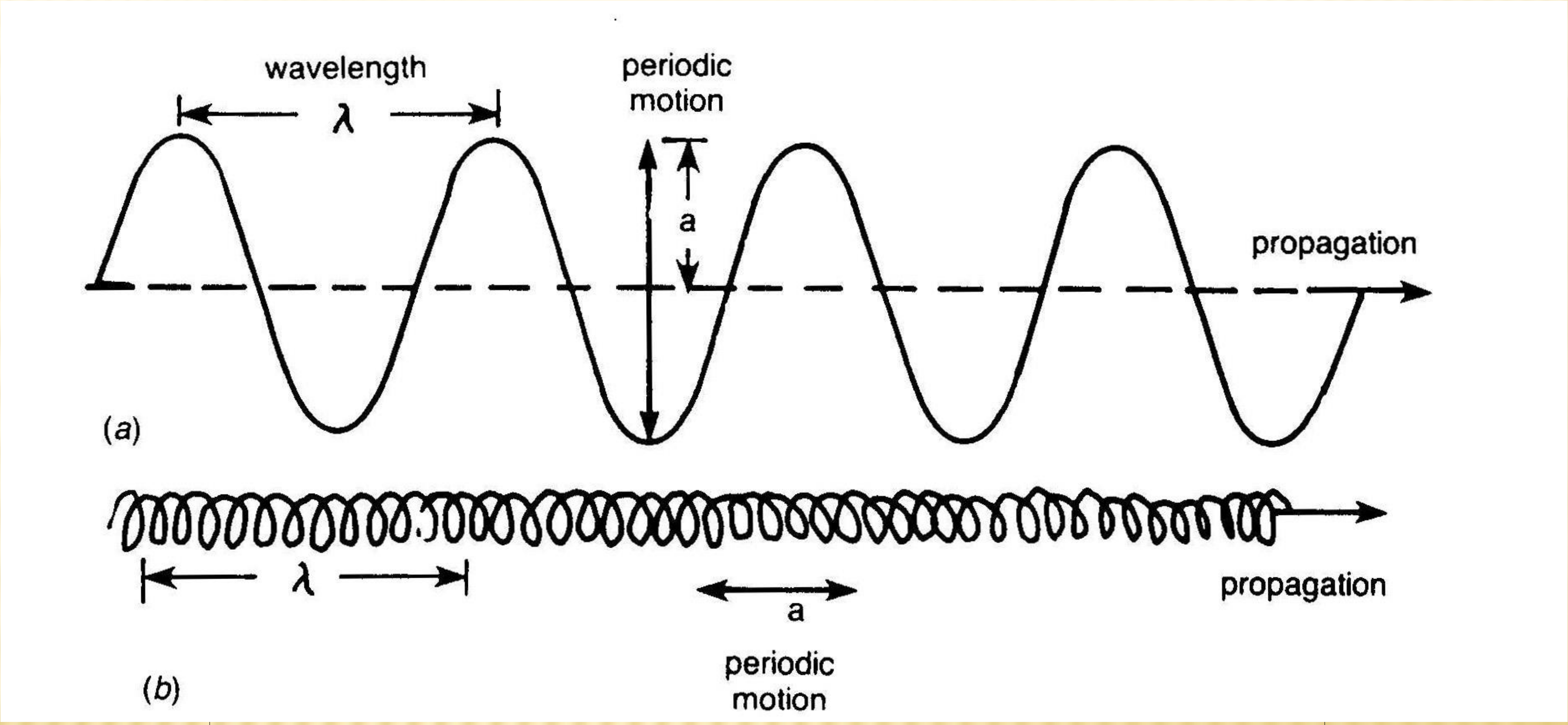
An important characteristic of wave motion is that the speed of propagation is determined by the medium in which the wave is traveling and usually does not depend upon the frequency or amplitude of the wave.

The wavelength **λ** is given by: $$\lambda = \frac{v}{f} = vT$$

The relationship applies to all types of waves: sound waves, water waves, light waves, X-rays, etc.

Since the speed remains constant for a given type of wave motion, an increase in frequency will result in a shorter wavelength.

The wavelength λ is the shortest repeat distance for the periodic wave, such as the distance between crests or the distance between compressions of the spring.


wavelength
λ
periodic
motion
a
propagation
(a)
λ
a
propagation
periodic
motion
(b)

WAVE PROPERTIES OF SOUND AND LIGHT

Def. Sound - a traveling pressure wave which may be propagated through a material medium.

It cannot travel through a vacuum.

“Audible" sound – f=20 ÷ 20000 Hz (though the range of human hearing varies widely).

“Ultrasonic" sound – f > 20000 Hz

“Infrasonic" sound – f < 20 Hz

Unless otherwise indicated, sound will be taken to mean audible sound, that is, periodic pressure variations with frequencies in the audible range.

The phenomenon of pitch perception is basically one of measuring the frequency of an incoming sound wave, a higher frequency being perceived as a higher pitch.

Sound waves originate when some elastic object vibrates back and forth rapidly enough to send an audible frequency wave through the medium in which it vibrates. A good example is the tuning fork.

Def. Light - a traveling wave consisting of propagating electric and magnetic fields.

Transverse wave, $v = 3 \times 10^8$ m/s.

The perceived color of light is basically a measurement of its frequency.

Red light corresponds to the lowest visible frequency. Increasing frequencies correspond to the successive colors of the rainbow spectrum until the highest visible frequency, violet, is reached. The color of light is analogous to the pitch of sound (both are determined by frequency).

Infrared and ultraviolet light – waves with frequencies below and above the visible light.

ENERGY IN WAVES

All traveling waves carry with them a certain amount of energy.

Consider a small sound source in air which radiates sound in all directions. The sound source produces a certain amount of power.

Since the power at a given distance from the source is spread over a large area, it is convenient to use the power per unit area rather than the total power.

The power per unit area is the *intensity* of the radiation

$$I = \frac{P}{A} \ [\frac{W}{m^2}]$$

If the source is very small and emits its energy uniformly in all directions, it spreads its energy in a spherical distribution. The intensity measured at a distance *r* from the source will then be

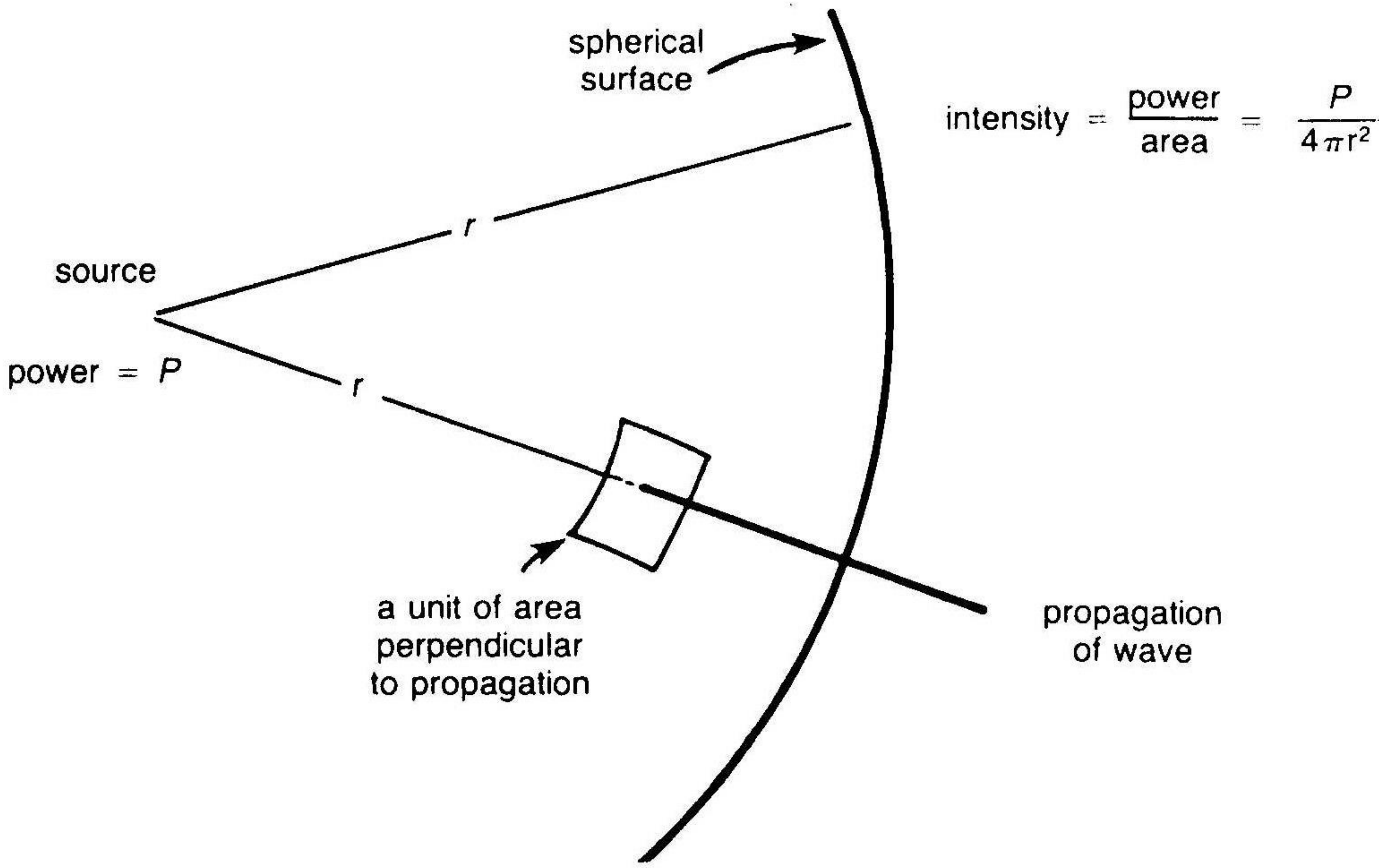
$$I = \frac{P}{4\pi r^2}$$

This intensity relationship is known as the *inverse square law.*

Most things in nature proceeding from a point source to all directions are characterized by an inverse square law relationship to the distance from the source: for example, electric field from a point charge, gravity field and sound intensity from a point source.

Thus I will drop off by a factor of 4 when the distance *r* is doubled, by a factor of 9 when the distance is tripled, and so forth. Cutting the distance to the source in half raises the light intensity by a factor of four.

This inverse square law also applies to other types of wave motion such as γ-rays and x-rays when they are spherically distributed.


spherical surface
intensity = $\frac{\text{power}}{\text{area}} = \frac{P}{4\pi r^2}$
r
source
power = P
r
a unit of area perpendicular to propagation
propagation of wave

INTERFERENCE AND STANDING WAVES

Interference refers to the addition of two or more waves which pass the same point in space.

If water waves from two different sources meet on the surface of a lake, there will be places where the crest from one set of waves will coincide with a trough from the other. If the amplitudes of the waves are equal, they will completely cancel at this point, leaving a momentary flat spot on the lake surface. This is an example of *destructive interference.*

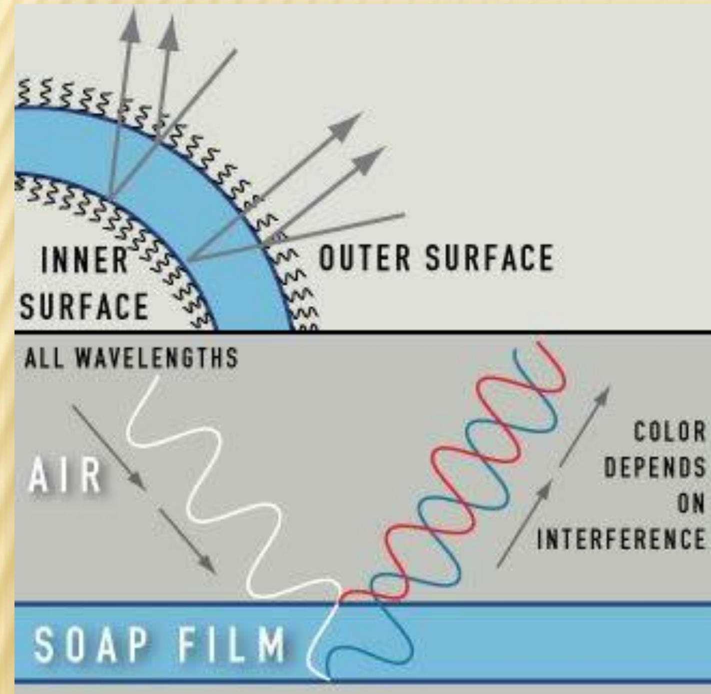
Constructive interference occurs where two wave crests coincide and add to produce a larger crest.

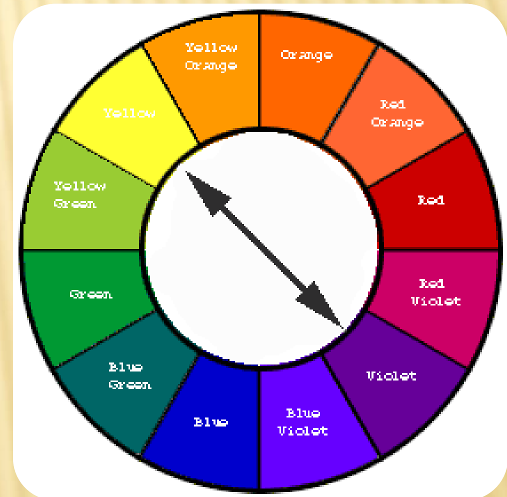
An example of the interference of sound waves is the phenomenon of "beats" when two musical tones near the same frequency are sounded. As the two waves of different λ reach the ear, their crests will coincide at one instant and then oppose each other a short time later.

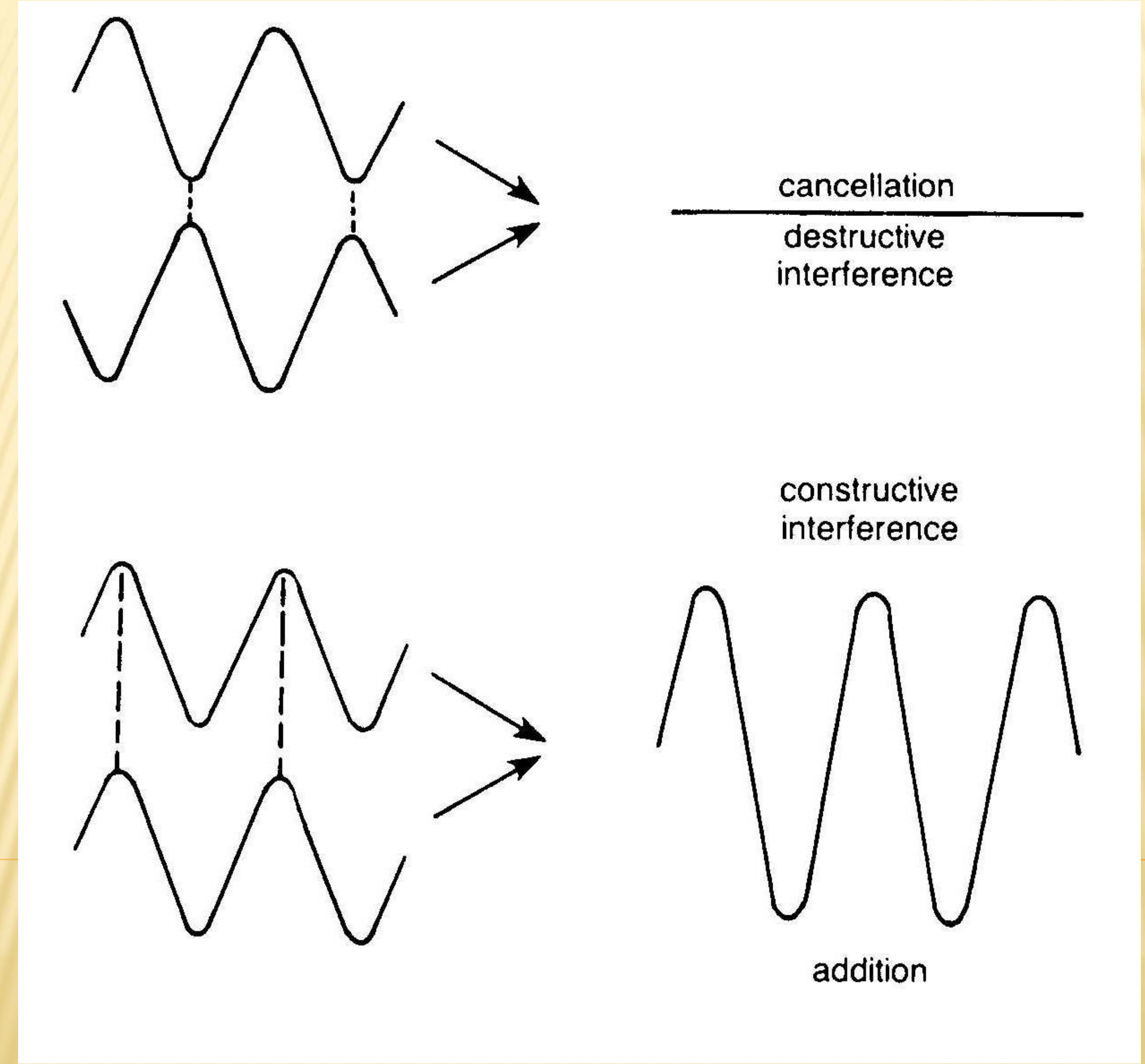
The result is a periodic rising and falling of the sound intensity, which occurs at a rate equal to the difference between the two frequencies.

E.g. If f_1=440 cycles/s and f_2=442 cycles/s are sounded simultaneously, the sound intensity will rise and fall twice per second.

Interference between light waves is responsible for the rainbow-like colors seen in soap bubbles and thin oil films. A portion of the light striking a thin soap film will reflect from the top surface, and a portion will enter the soap and reflect from the bottom surface. Depending on λ (color), the two reflected waves may interfere constructively or destructively. Since white light is a mixture of all λ, the removal of one λ by destructive interference will remove one color from the white light, leaving the complementary color as the apparent color of the film at this point.





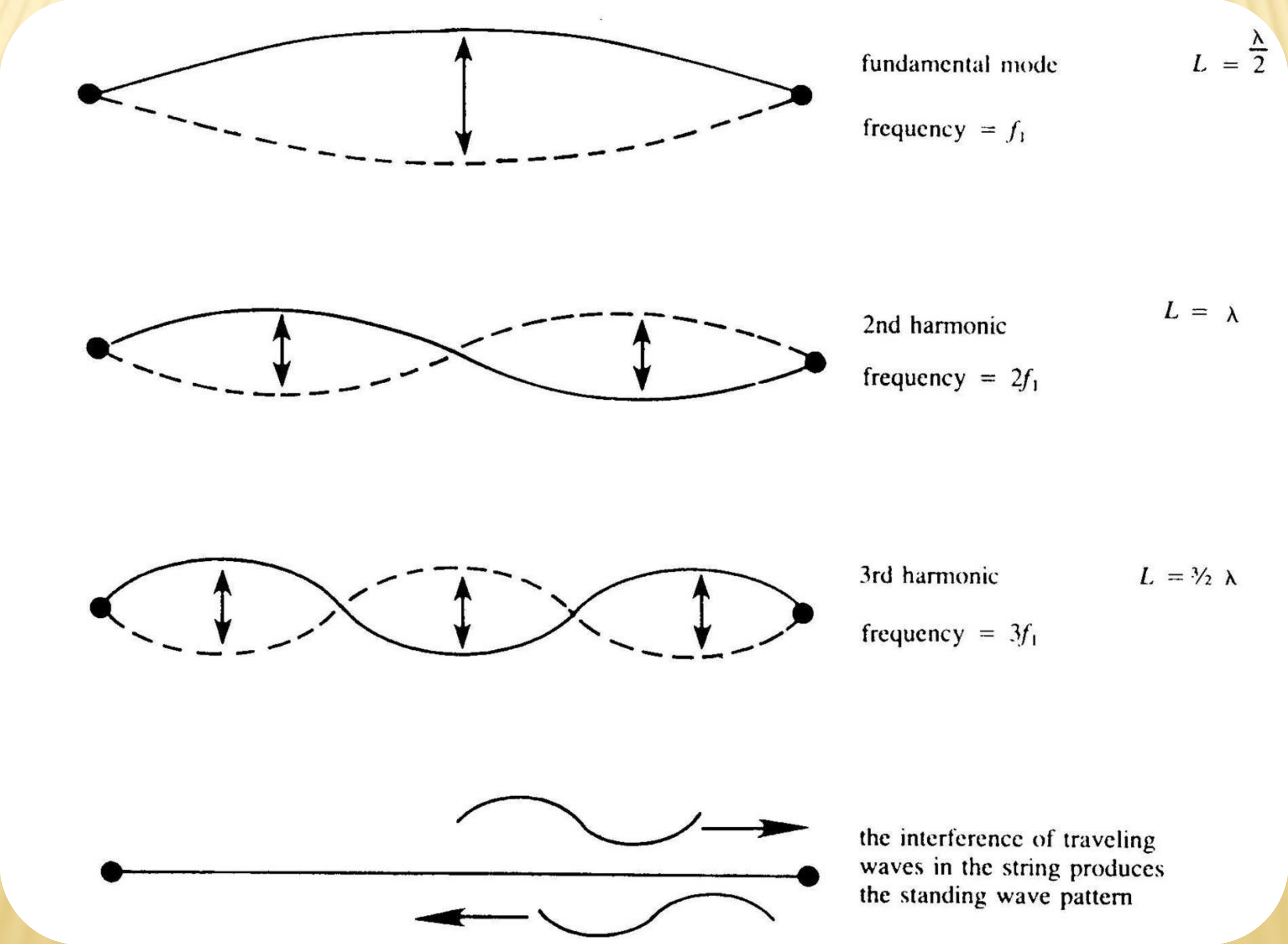

cancellation
destructive
interference
constructive
interference
addition

When a wave is produced in an elastic medium like a stretched guitar string, the wave will reflect at the end of the string and return. The reflected wave may interfere constructively or destructively with the incident wave.

When the length of the string = a half-wavelength or an integer multiple of half-wavelengths ($\lambda/2$, $3\lambda/2$, 2λ, $5\lambda/2$, etc.) then the interference between reflected waves will cause a *standing wave* - the string will vibrate up and down in fixed segments.

λ satisfying the standing wave condition will tend to be sustained by the string while other λ will quickly die away.

The frequencies corresponding to the standing waves are said to be natural frequencies or "resonant" frequencies.



THE DOPPLER EFFECT

The Doppler effect is based on the fact that a sound wave consists of a series of high and low pressure areas in the air which propagate with

$v = 345$ m/s at $T = 20°C$.

Suppose that a fixed sound source emits a sound with $f = 345$ Hz, so that there is $\lambda = 1$ m between compressions in the air.

Any stationary observer would perceive $f = 345$ Hz.

Now suppose the sound source is moving away from the observer with a speed of 34.5 m/s.

During one full T, an air compression would have moved outward 1 m, but the sound source would have moved 0.1 m farther away from the observer. This would cause the next compression to be 1.1 m away from the previous one, giving an effective λ= 1.1 m instead of 1.0 m. Since the sound v=345 m/s, depends only upon the medium (air), the observer will receive the frequency f=314 Hz.

Now if the sound source was traveling toward the observer at 34.5 m/s, the source would move 0.1 m closer to the observer between each compression.

The observed sound would be raised in frequency (pitch) to f_0=383 Hz.

The amount of pitch drop is proportional to the speed of the sound source and could be used to measure its speed. This speed-dependent pitch drop explains why the sound of a fast aircraft changes from a high pitched whine to a low thundering roar as it passes overhead.

The Doppler effect for a moving sound source can be summarized by the equation

$$\frac{f_{observed}}{f_{source}} = \frac{v}{v \pm v_s}$$

where v is the speed of sound and v_s is the source speed. The minus sign is used for an approaching sound source and the positive sign for a receding source.

Similar Doppler shifts are observed when the sound source is stationary and the observer is moving. When either the source or the observer or both may be moving with respect to the medium, the generalized Doppler equation is:

$$\frac{f_{observed}}{f_{source}} = \frac{v \pm v_o}{v \pm v_s}$$

Doppler shifts of light frequencies enable astronomers to measure the speed of recession of distant stars.

Doppler shifts of the frequency of microwave radiation (radar) enable air traffic controllers to measure the speed of approaching aircraft .

ULTRASONIC SOUND

Ultrasound – f > 20kHz without a definite upper limit.

Frequencies as high as 15 MHz are routinely used in medical applications.

Most of the diagnostic uses of ultrasonic sound make use of echo techniques analogous to the SONAR echolocation of underwater objects by submarines and fishing vessels. SONAR - SOund Navigation And Ranging.

An echo is a reflection of a sound wave from some interface where the nature of the medium in which the sound travels undergoes a significant change.

Measurement of the time elapsed before the echo is received and a knowledge of the speed of sound in the medium make possible an accurate range determination.

When ultrasonic waves are directed into the body, reflections occur at interfaces between different tissues or fluids. A reflection will occur at any interface when the speed of sound changes.

The speed of sound in tissue is primarily dependent upon density, so the outlines produced are largely outlines of density changes.

The minimum size of a resolved object depends upon the wavelength of the sound; the sound wavelength should be considerably smaller than the object in order to visualize it clearly.

The same principle applies to vision. We see ordinary objects with clear detail because the light wavelengths which are reflected to our eyes are much smaller than the objects seen.

When 15 MHz sound waves are used, the wavelength is about 0.1 mm, so the sound wavelength is then not a limiting factor.

However, the higher the frequency the more the transmission of the ultrasound is attenuated.

A compromise must be made between resolution of the image and the depth of penetration of the scan.

For the eye examination, f≈15 MHz can be used,
for abdominal examinations , f=1 - 3 MHz.

Present techniques involve a series of short sound pulses, with the echoes received and displayed on an oscilloscope.

Cross-sectional views of the abdomen may be obtained with two-dimensional scanners. The ultrasonic beam is moved back and forth across the abdomen in a given plane, while the position of the probe is registered and stored so that a two-dimensional image can be constructed.

Ultrasonic scanners making use of the Doppler effect have been used:

- to study the function of the heart and the blood flow through major arteries,
- for the early detection of fetal heart tones.

The Doppler shifted echoes from moving objects are at a slightly different frequency than the echoes from fixed objects. The echo sounds from fixed objects are at precisely the same f as the source.

1. One possible scheme for using these facts is to filter out all the echoes at the same f as the source, so that to detect only the Doppler shifted echoes – such a scheme is difficult in practice.

2. If you detect a sound signal at 40 kHz and at the same time a signal at 40,2 kHz, these signals will interfere with each other, canceling each other at one instant and adding to each other at another. The effect is to produce a "beat frequency" signal at 200 Hz, the difference between the two frequencies.

If the aim is to study only the moving parts, then the detectors can be set for the low frequency beat signals and both high frequencies can be "thrown away" as no longer useful.

E. g., the ultrasound frequencies in a Doppler pulse detector may be in the MHz range, but the beat frequencies with the Doppler shifted echoes off the moving blood in the arteries may be in the ordinary sound range of 100 Hz so that they can be picked up by a sensitive microphone.

In vascular studies both the sources and the detectors of the ultrasound are called transducers.

A transducer is a device that converts energy from one form to another.

source transducer: electrical energy →ultrasound

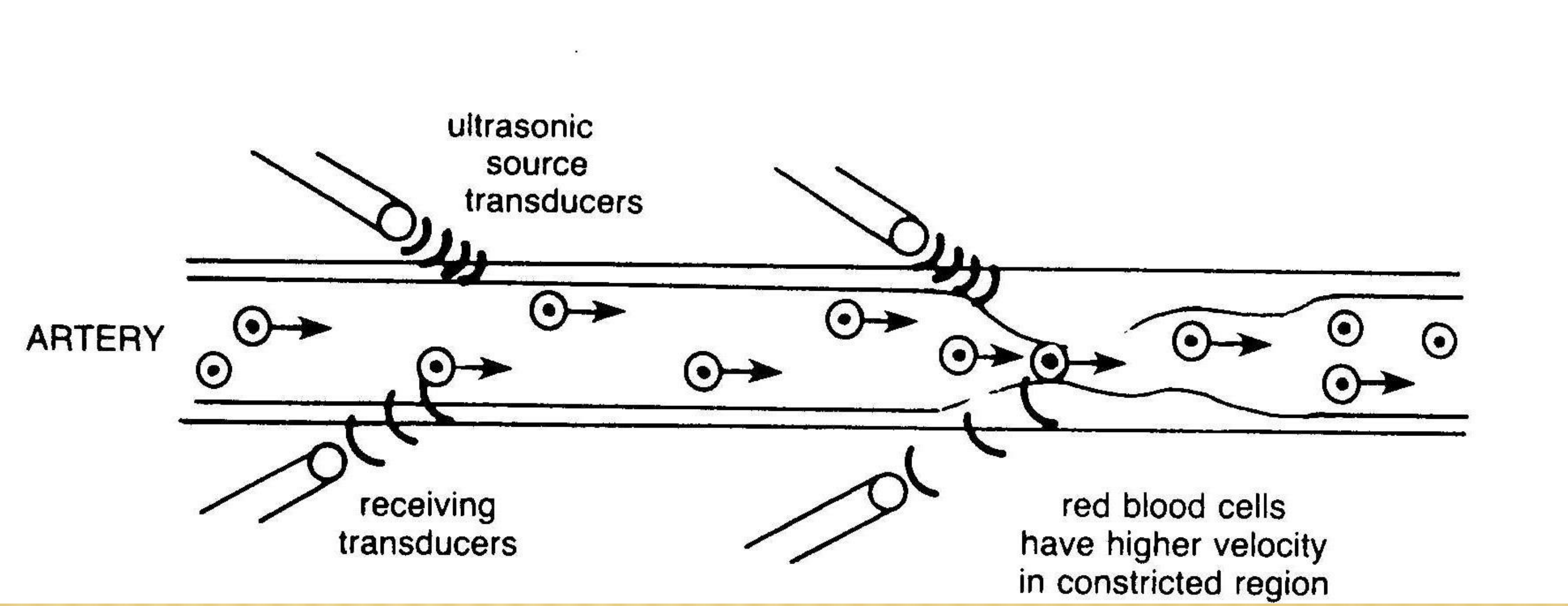
receiving transducer : ultrasound →electrical signal

The ultrasound from the source transducer produces Doppler shifted echoes off the moving red blood cells in the arteries.

An echo signal with a frequency that depends upon the speed of the blood flow is registered.

If there is a marked narrowing or stenosis of the artery at some point, the speed of the blood will increase at that point.

Because the beat frequency is directly proportional to the speed of the reflecting objects, the Doppler probe provides a sensitive tool for locating constrictions in the vascular system.



REVIEW QUESTIONS

1. A characteristic of elastic media is the fact that they have a definite equilibrium configuration at which they eventually come to rest after being disturbed. Why do they not immediately come to rest after being disturbed instead of oscillating back and forth through the equilibrium point?

2. Give some examples of interference of waves.

3. Can sound waves travel through a vacuum?

4. Does changing the frequency of a traveling wave also change its speed of propagation?

5. What is the difference between a longitudinal and a transverse wave? Name some examples of each.

7. What information can you get from Doppler ultrasound techniques that is not available from ordinary ultrasound scans?